

## ABSTRACT

Recently, many novel concepts originated in dynamical systems or information theory have been developed, partly motivated by specific research questions linked to geosciences, and found a variety of different applications. This continuously extending toolbox of nonlinear time series analysis highlights the importance of the dynamical complexity to understand the behavior of the complex Earth's system and its components. Here, we propose to apply such new approaches, mainly a series of entropy methods to the time series of the Earth's magnetic field measured by the Swarm constellation. Swarm is an ESA mission launched on November 22, 2013, comprising three satellites at low Earth polar orbits. The mission delivers data that provide new insight into the Earth's system by improving our understanding of the Earth's interior as well as the near-Earth electromagnetic environment. We show successful applications of methods originated in information theory to quantitatively studying complexity in the dynamical response of the topside ionosphere, at Swarm altitudes, to space weather events.

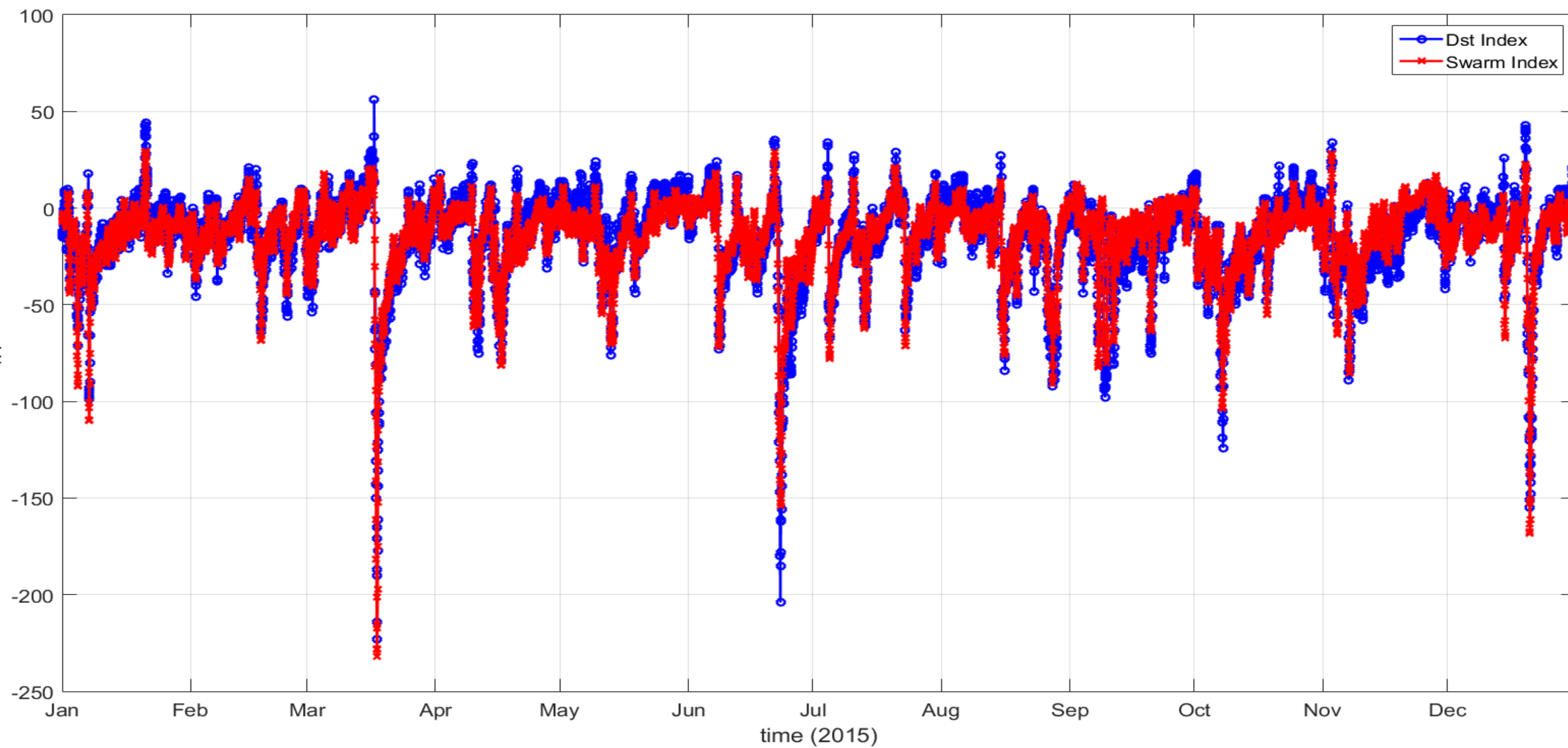


Figure 1: Dst index (blue) and Swarm Dst-like index (red) for 2015. The estimation of Pearson's Correlation Coefficient between Dst index and Swarm Dst-like index for the entire 2015 time series provided values over 0.90 for a wide range of values for the free parameters.

## Wavelets [Balasis et al., 2006]

$\beta$  exponent and its relation to Hurst:  $\beta=2H+1$ , where  $H$  is the Hurst exponent.

- The exponent  $H$  characterizes the persistent/anti-persistent properties of the signal. The range  $0 < H < 0.5$  ( $1 < \beta < 2$ ) during the normal period indicates anti-persistence, reflecting that if the fluctuations increase in a period, it is likely to decreasing in the interval immediately following and vice versa.
- We pay attention to the fact that the time series appears persistent properties,  $0.5 < H < 1$  ( $2 < \beta < 3$ ). This means that if the amplitude of fluctuations increases in a time interval it is likely to continue increasing in the interval immediately following.
- $H=0.5$  ( $\beta=2$ ) suggests no correlation between the repeated increments. It marks the transition between persistent and anti-persistent behavior in the time series.

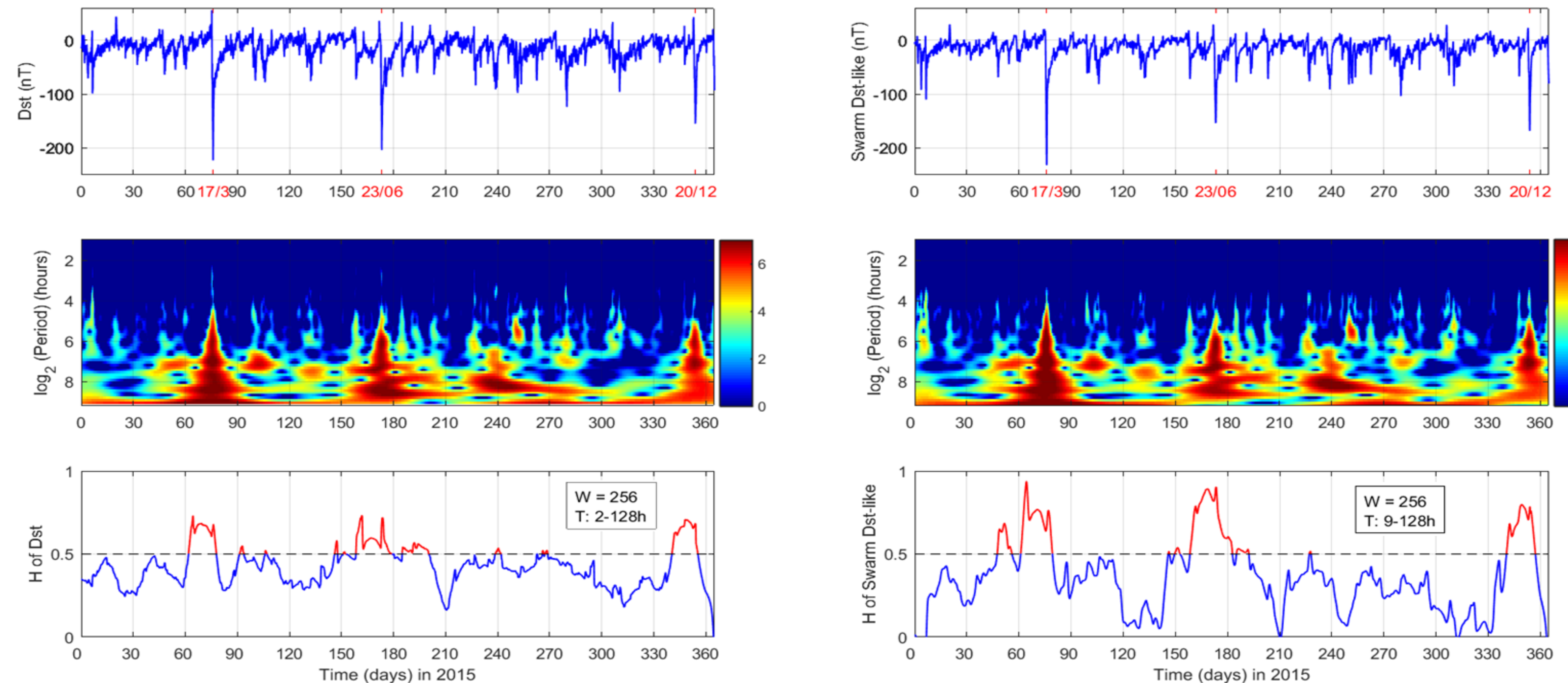


Figure 2: Wavelet Spectral Analysis & Temporal Variation of the  $H$  index, for Dst and Swarm Dst-like index, respectively.

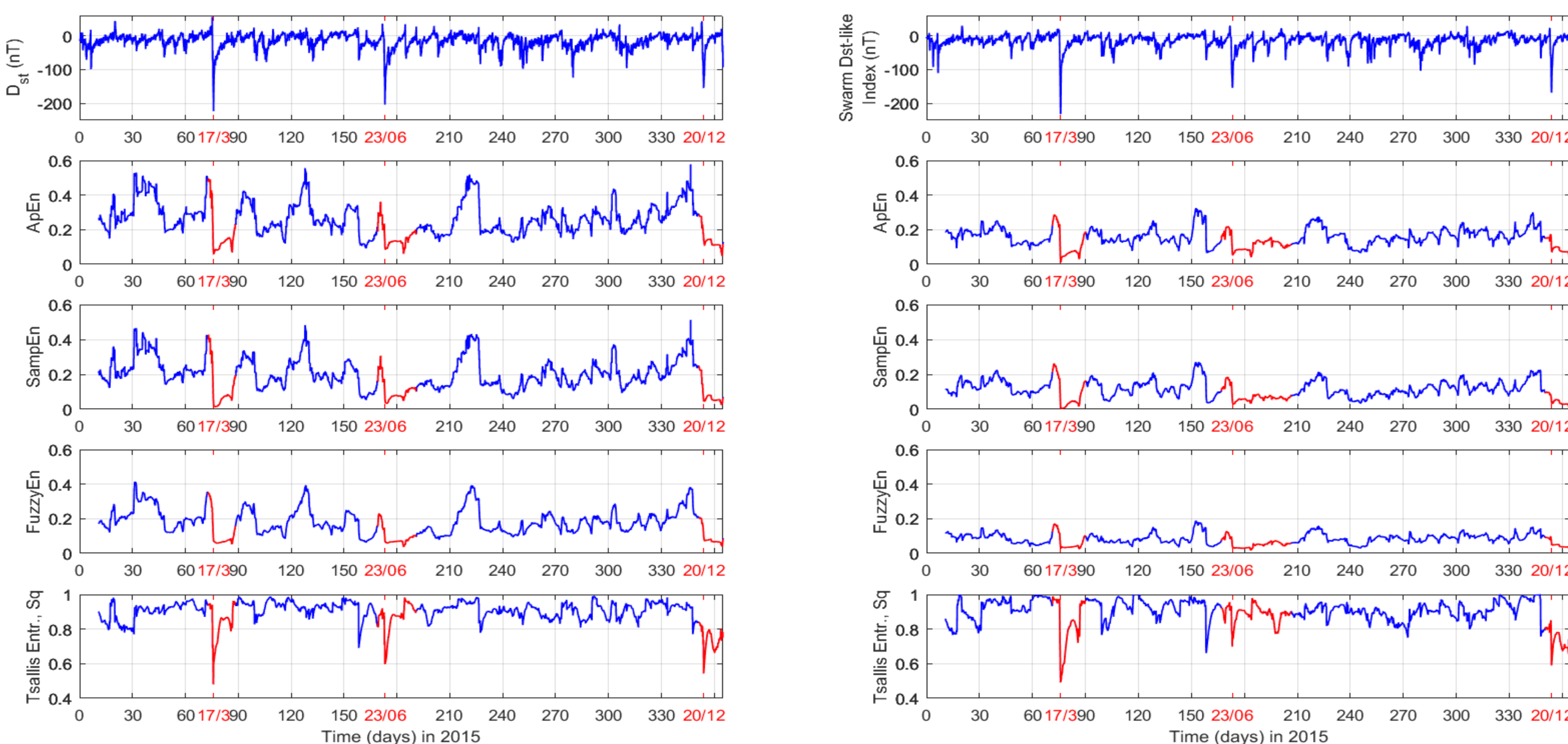


Figure 3: Entropies (ApEn, SampEn, FuzzyEn, Tsallis) for Dst index and Swarm Dst-like index, respectively.

## ACKNOWLEDGEMENTS

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## Swarm derived Dst index [Balasis et al., 2019]

- Extract Total Magnetic Field Series from MAG\_LR (1 Hz) product (Swarm-A)
- Subtract CHAOS-6 [Finlay et al., 2016] Internal Field Model
- Remove values that lie above  $\pm 40^\circ$  in Magnetic Latitude
- Remove spikes and interpolate small data gaps
- Apply a low-pass Chebyshev Type I filter with a cutoff period of 13 hours
  - A 12-hour averaging provides complete global coverage!
- Remove seasonal effects and the Local Time drift of the satellites' orbit
  - Use a Chebyshev Type I filter with a cutoff period of approx. 4 months to model this slowly varying component
  - Subtract it from the filtered series of step 5
- Apply a linear transform to get the Swarm Index:

$$S_{index} = 2.5 B_{(6)} - 15$$

## Entropies [Balasis et al., 2008, 2009, 2013, 2016]

- Approximate entropy (ApEn) has been introduced by Pincus as a measure for characterizing the regularity in relatively short and potentially noisy data. More specifically, ApEn examines time series for detecting the presence of similar epochs; more similar and more frequent epochs lead to lower values of ApEn.
- Sample entropy (SampEn) was proposed by Richman and Moorman as an alternative that would provide an improvement of the intrinsic bias of ApEn.
- Fuzzy entropy (FuzzyEn), like its ancestors, ApEn and SampleEn, is a "regularity statistic" that quantifies the (un)predictability of fluctuations in a time series. For the calculation of FuzzyEn, the similarity between vectors is defined based on fuzzy membership functions and the vectors' shapes. FuzzyEn can be considered as an upgraded alternative of SampEn (and ApEn) for the evaluation of complexity, especially for short time series contaminated by noise.
- Inspired by multi-fractal concepts, Tsallis [1988, 1998] has proposed a generalization of the Boltzmann-Gibbs statistics, i.e. the Tsallis Entropy,  $S_q$ .

## CONCLUSIONS

- The newly proposed Swarm Dst-like index monitors magnetic storm activity at least as good as the standard Dst index.
- The information-theoretic measures for the Swarm Dst-like index seem to work at least as good as for Dst and in some cases even better.
- The new 1 Hz Swarm derived Dst index is promising for space weather forecasting.